Advances in Plastic Anisotropy and Forming Limits in Sheet Metal Forming

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In the last decades, numerical simulation has gradually extended its applicability in the field of sheet metal forming. Constitutive modeling and formability are two domains closely related to the development of numerical simulation tools. This paper is focused, on the one hand, on the presentation of new phenomenological yield criteria developed in the last decade, which are able to describe the anisotropic response of sheet metals, and, on the other hand, on new models and experiments to predict/determine the forming limit curves. [DOI: 10.1115/1.4033879]

Anisotropic Yield Criteria

The accuracy of the simulation results is given mainly by the accuracy of the material model. In the last years, scientific research has been oriented toward the development of new material models that are able to describe the material behavior (mainly the anisotropic one) as accurately as possible [1–11]. The computer simulation of the sheet metal forming processes needs a quantitative description of the plastic anisotropy by the yield locus.

For the case of an isotropic metallic material, the well-known von Mises yield criterion is often sufficient to describe yielding. This is, however, not true for anisotropic materials, especially aluminum sheet metals. In order to take into account anisotropy, the classical yield criterion proposed by von Mises should be modified by introducing additional parameters. A simple approximation for the case of normal anisotropy is given by the quadratic criterion of Hill [12]

$$\sigma_1^2 - \frac{2r}{1+r}\sigma_1\sigma_2 + \sigma_2^2 = \sigma_u^2 \qquad (1)$$

where r is the normal anisotropy coefficient and σ_u is the uniaxial in-plane yield stress.

Woodthrope and Pearce [13] have found that the yield stress in balanced biaxial tension, σ_b, for aluminum alloy sheets having a r-value lying between 0.5 and 0.6 is significantly higher than the uniaxial yield stress in the plane of the sheet. However, Hill's quadratic criterion [12] cannot describe this behavior, i.e., materials with $r < 1$ and $\sigma_b > \sigma_u$. To capture this so-called "anomalous" behavior, nonquadratic yield formulations were considered [13].

Later on, several scientists have proposed more and more sophisticated yield functions for anisotropic materials. Hill [14] himself improved his criterion and proposed a nonquadratic form. Although the anomalous behavior is captured with this function, the predicted yield surfaces are sometimes different from those either determined experimentally or predicted with polycrystalline models. Hill [15] included the shear stress component in the expression of anisotropic yield function. Hill [16] stated that none of the previous criteria is able to represent the behavior of a material exhibiting a tensile yield stress almost equal in value in the rolling and transverse direction, while r-values vary strongly with the angle to the rolling direction. Another important research direction in the field was initiated by Hershey [17], who introduced a nonquadratic yield function for isotropic materials, based on the results of polycrystalline calculations. This criterion was later generalized to anisotropic materials by Hosford [18]. This criterion is a particular expression of Hill's 1979 yield criterion. Its main advantage is that it leads to a good approximation of yield loci computed using the polycrystalline Bishop–Hill model by setting $a = 6$ for BCC materials and $a = 8$ for FCC materials [19]. An important drawback of this as well as of Hill's nonquadratic yield criteria is that they do not involve shear stresses. Barlat and Lian [20] successfully extended Hosford's 1979 criterion to capture the influence of the shear stress and proposed the following yield function:

$$f = a|k_1 + k_2|^M + a|k_1 - k_2|^M + (2-a)|2k_2|^M = 2\sigma_e^M \qquad (2)$$

where

$$k_1 = \frac{\sigma_x + h\sigma_y}{2}; \quad k_2 = \left[\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + p^2\tau_{xy}^2\right]^{1/2} \qquad (3)$$

while a, h, p, and M are material parameters.

Different other nonquadratic formulations were developed: Gotoh [21] introduced a fourth-degree polynomial yield function; Budiansky [22] prescribed a parametric expression in polar coordinates of the yield function (extended by Tourki et al. [23]). Barlat et al. [24] developed a six-component yield function, by using a linear transformation of the stress state (extended successively by Barlat et al. [25], denoted as Yld94 and by Barlat et al. [26], denoted as Yld96). Karafillis and Boyce [27] proposed a general yield criterion using a "weighted" linear transformation (extended by Bron and Besson [28]).

During the last years, new yield functions were introduced in order to improve the fitting of the experimental results, especially for aluminum and magnesium alloys. In order to remove the disadvantages of the Yld94 and Yld97 yield criteria, while still preserving their flexibility, Barlat in 2003 proposed [29] a new model particularized for plane stress (2D) (Yld 2000). The expressions of the two isotropic yield functions considered in Ref. [29] are

$$\phi'(\mathbf{s}) = |s_1 - s_2|^m, \quad \phi''(\mathbf{s}) = |2s_2 + s_1|^m + |2s_1 + s_2|^m \qquad (4)$$

leading to the resulting anisotropic yield function

$$\phi = \phi'(\mathbf{X}') + \phi''(\mathbf{X}'') = 2\bar{\sigma}^m \qquad (5)$$

where m is the Hosford's exponent, $\mathbf{s}$ is the deviatoric stress tensor, and $\mathbf{X}$ is the linearly transformed stress tensor.

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Table 1 The main FE commercial software and the anisotropic yield criteria implemented in them

	Hill 1948	Hill 1990	Barlat 1989	Barlat 2003	Vegter 1995	BBC 2005
ABAQUS	■	■	■	■		
AUTOFORM	■	■				■
LS-DYNA	■	■	■	■		
MARC	■	■				
PAM STAMP	■	■	■		■	

Using the following linear transformation on stresses

$$\begin{aligned}\mathbf{X}' &= \mathbf{C}'\mathbf{s} = \mathbf{C}'\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}'\boldsymbol{\sigma}\\ \mathbf{X}'' &= \mathbf{C}''\mathbf{s} = \mathbf{C}''\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}''\boldsymbol{\sigma}\end{aligned} \tag{6}$$

where $\mathbf{C}'$ and $\mathbf{C}''$ (or $\mathbf{L}'$ and $\mathbf{L}''$) represent the linear transformations and $\mathbf{T}$ is a matrix relating the deviatoric to the Cauchy stresses

$$\begin{bmatrix} X'_{xx} \\ X'_{yy} \\ X'_{xy} \end{bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{66} \end{bmatrix} \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

A similar expression with double prime defines $\mathbf{C}''$. A plane stress state can be described by the two principal values of $\mathbf{X}'$ and $\mathbf{X}''$

$$X_{1,2} = \frac{1}{2}\left(X_{xx} + X_{yy} \pm \sqrt{(X_{xx} - X_{yy})^2 + 4X_{xy}^2}\right) \tag{8}$$

with the appropriate indices (prime and double prime) for each stress.

The yield function is defined by eight coefficients, determined using as input the values of the stresses and anisotropy coefficients in tension along three directions, the balanced biaxial flow stress and biaxial anisotropy coefficient.

Barlat et al. [30] and Aretz and Barlat [31] proposed a generalization of Yld 2000 model for 3D case using 18 mechanical parameters. The implementation of the Barlat 2004-18p model in finite-element codes [32] is allowed, proving its capability to predict the occurrence of six and eight ears in the process of cup drawing.

To introduce orthotropy in the expression of an isotropic criterion, Cazacu and Barlat [33] proposed an alternative method based on the theory of the representation of tensor functions. The method is applied for the extension of Drucker's isotropic [34] yield criterion to transverse isotropy and cubic symmetries [35]. The experimental researches [36] have shown that for some HCP alloys (e.g., magnesium- and titanium-based alloys), the yield surface is better described by fourth-order functions. As a consequence, in order to describe such behavior, Cazacu et al. [37] proposed the model of an isotropic yield function for which the degree of homogeneity is not fixed.

Vegter [38,39] proposed the representation of the yield function with the help of Bezier's interpolation using directly the test results (pure shear point, uniaxial point, plain strain poin,t and equi-biaxial point).

The analytical expression of the Vegter yield function is

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = (1-\lambda)^2 \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^r + 2\lambda(1-\lambda)\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_i^h + \lambda^2 \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_{i+1}^r \tag{9}$$

for σ_e and angle φ where

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}_{i+1}^r = \sum_{j=0}^{mcos} \begin{pmatrix} d_1^j \\ d_2^j \end{pmatrix}_i^r \cos(2j\varphi) \tag{10}$$

is a trigonometric expansion associated to the reference point

$$R(\varphi) = \sum_{j=0}^{mcos} b^j \cos(2j\varphi) \tag{11}$$

is the cosine interpolation of the function $R(\varphi)$; φ is the angle between the principal directions and the orthotropic axes; λ is a parameter of the Bézier function; r is a superscript denoting the reference point; h is a superscript denoting the breaking point; $\begin{pmatrix} d_1^j \\ d_2^j \end{pmatrix}_i^r$ are parameters of the trigonometric interpolation to be determined at the reference points; and b^j are parameters of the trigonometric interpolation of the R-function.

Hill [40] proposed in 1950 a general formulation of a plane-stress anisotropic yield criterion having the polynomial expression. Gotoh [21] succeeded to apply that idea in the 1970s by developing a polynomial yield function of fourth degree. During the last years, a new family of polynomial yield criteria has been created on the basis of Hill's idea by Comsa and Banabic [41]. Soare et al. [42] proposed three yield criteria expressed by polynomial functions of fourth, sixth, and eighth orders, respectively (poly 4, poly 6, and poly 8).

Yld 2000 [29], Vegter et al. [38], and BBC 2005 [43] models have been implemented in the last decade in the main FE commercial softwares (see Table 1).

Table 2 presents the main yield criteria developed for description of the anisotropic plastic behavior. The mechanical parameters used for the identification of the models are also presented. The following notations have been used in the table: 3D, criterion can be extended to spatial stress states; $A1$ and $A2$, criterion can describe the first- and second-order anomalous behavior (see more details in Ref. [6]).

The CERTETA team has developed several anisotropic yield criteria. A description of these developments is presented in the next section.

Advanced Yield Criteria Developed in the CERTETA Research Center

CERTETA is a Romanian research center that supports metal forming companies in developing advanced and efficient technologies (see more details in the CERTETA webpage[1]). In 2000, the members of CERTETA started a research program having as principal objective the development of a model that is able to provide an accurate description of the yield surfaces predicted by texture computations. The new formulation [44,45] was developed on the basis of the formulation proposed by Barlat in 1989 [20]. By adding weight coefficients to that model, the researchers succeeded to develop a flexible yield criterion. The version published in 2005 [43] incorporates a number of eight coefficients and, consequently, its identification procedure uses eight mechanical parameters (three uniaxial yield stresses, three uniaxial coefficients of anisotropy, a biaxial yield stress, and a biaxial coefficient of plastic anisotropy). (An improvement of this criterion has been implemented in the finite-element commercial code AUTOFORM version 4.1 [46]. The equivalent stress is defined by the following formula:

[1] http://certeta.utcluj.ro/

Table 2 The main yield criteria and experimental data to be evaluated for their material parameters' identification [6]

Author, year	σ0	σ30	σ45	σ75	σ90	σb	r0	r30	r45	r75	r90	rb	3D	A1	A2
Hill's family															
Hill, 1948	X						X		X		X		X		
Hill, 1979	X					X	X						X	X	
Hill, 1990	X		X		X	X			X					X	
Hill, 1993	X				X	X	X				X			X	X
Lin and Ding, 1996	X				X	X	X		X		X			X	X
Hu, 2005	X		X		X	X	X		X		X		X	X	X
Leacock, 2006	X		X		X	X	X		X		X			X	X
Hershey's family															
Hosford, 1979	X						X				X		X	X	
Barlat, 1989	X						X				X			X	
Barlat, 1991	X		X		X	X							X	X	
Karafillis and Boyce, 1993	X		X		X		X		X		X		X	X	X
Barlat, 1997	X		X		X	X	X		X		X		X	X	X
Banabic, 2000	X		X		X	X	X		X		X		X	X	X
Barlat, 2000	X		X		X	X	X		X		X			X	X
Bron and Besson, 2003	X		X		X	X	X		X		X	X	X	X	X
Barlat, 2004	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Banabic, 2005	X		X		X	X	X		X		X	X	X	X	X
Banabic, 2008	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Aretz and Barlat, 2012	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Drucker's family															
Cazacu-Barlat, 2001	X	X	X	X	X	X	X	X	X	X	X		X	X	X
Cazacu-Barlat, 2003	X	X	X	X	X	X	X	X	X	X	X		X	X	X
Cazacu-Pluncket, 2006	X	X	X	X	X	X	X	X	X	X	X		X	X	X
Polynomial criteria															
Gotoh, 1977	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Comsa and Banabic, 2007	X		X		X	X	X		X		X	X	X	X	X
Soare, 2007 (Poly 4, 6, 8)	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Other criteria															
Ferron, 1994	X		X		X		X		X		X			X	
Vegter, 1995	X	X	*X*	*X*	X	X	X	X	X	X	X			X	X

$$\bar{\sigma} = \left[a(\Lambda+\Gamma)^{2k} + a(\Lambda-\Gamma)^{2k} + b(\Lambda+\Psi)^{2k} + b(\Lambda-\Psi)^{2k}\right]^{\frac{1}{2k}} \tag{12}$$

here $k \in \Im^{\geq 1}$ and $a,\ b > 0$ are material parameters, while Γ, Λ, and Ψ are functions depending on the planar components of the stress tensor

$$\begin{aligned} \Gamma &= L\sigma_{11} + M\sigma_{22} \\ \Lambda &= \sqrt{(N\sigma_{11} - P\sigma_{22})^2 + \sigma_{12}\sigma_{21}} \\ \Psi &= \sqrt{(Q\sigma_{11} - R\sigma_{22})^2 + \sigma_{12}\sigma_{21}} \end{aligned} \tag{13}$$

Nine material parameters are involved in the expression of the BBC equivalent stress: k, a, b, L, M, N, P, Q, and R (see Eqs. (12) and (13)). The integer exponent k has a special status, due to the fact that its value is fixed from the very beginning in accordance with the crystallographic structure of the material: $k=3$ for BCC materials; $k=4$ for FCC materials. The identification procedure calculates the other parameters (a, b, L, M, N, P, Q, and R) by forcing the constitutive equations associated to the BBC yield criterion to reproduce the following experimental data: the uniaxial yield stresses associated to the directions defined by 0 deg, 45 deg, and 90 deg angles measured from rolling direction (RD) (denoted as Y_0, Y_{45}, and Y_{90}); the coefficients of uniaxial plastic anisotropy associated to the directions defined by 0 deg, 45 deg, and 90 deg angles measured from RD (denoted as r_0, r_{45}, and r_{90}); the biaxial yield stress associated to RD and transversal direction (TD) (denoted as Y_b); and the coefficient of biaxial plastic anisotropy associated to RD and TD (denoted as r_b) (see more details in Ref. [6]).

The BBC 2005 model can be reduced to Hill 1948 or Barlat 1989 yield criteria, if they choose appropriate values of the material parameters (see more details in Ref. [6]).

The yield criterion proposed by Barlat and Lian in 1989 can be obtained by enforcing the following constraints on the material parameters:

$$Y = Y_0, \quad k = 3 \text{ or } 4, \quad L = N = Q, \quad M = P = R \tag{14}$$

The identification procedure needs only r_0, r_{45}, and r_{90} as input data.

Another situation of practical interest is the so-called normal anisotropy ($r_0 = r_{45} = r_{90} = r$, $Y_0 = Y_{45} = Y_{90} = Y$). In this case, BBC 2005 also reduces to the Hill 1948 or Barlat 1989 yield criteria (depending on the value of the exponent k)

$$\begin{aligned} &k = 1 (\text{Hill } 1948), \quad k = 3 \text{ or } 4\ (\text{Barlat } 1989), \\ &a = \frac{1}{1+r}, \quad b = \frac{r}{1+r}, \quad L = N = Q = M = P = R = \frac{1}{2} \end{aligned} \tag{15}$$

Figure 1 shows a comparison of the yield loci predicted by different formulations of BBC2005 for AA6016-T4 aluminum alloy. The mechanical parameters of the tested alloy are the following: $\sigma_0 = 139$ MPa, $\sigma_{45} = 137$ MPa, $\sigma_{90} = 136$ MPa, $\sigma_b = 140.76$ MPa, $r_0 = 0.724$, $r_{45} = 0.547$, $r_{90} = 0.602$, $r_b = 1.05$. Three experimental points are also plotted on the same diagram. Due to the fact that both BBC2005 with seven and eight coefficients used in identification procedure of the experimental value of $\sigma_b^{\exp}$, the predictions of these formulations are more accurate. The presented results show the ability of the BBC2005 yield criterion to provide an accurate description of the anisotropic behavior for AA6016-T4 aluminum alloy.

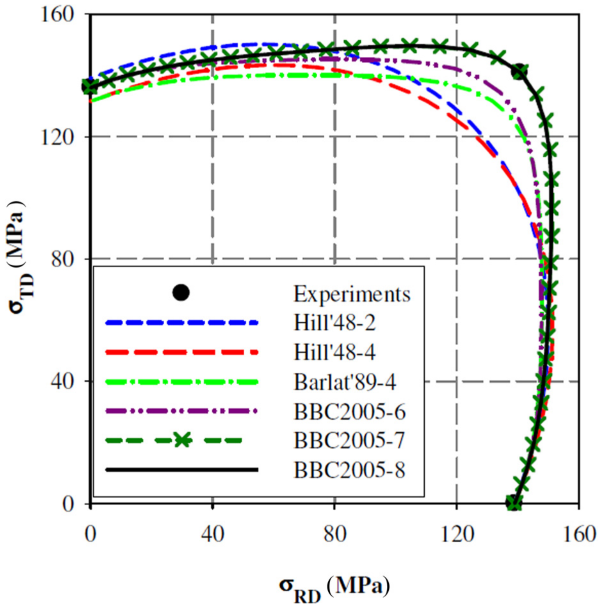


Fig. 1 Yield loci predicted by using different versions of the BBC2005 model for AA6016-T4 aluminum alloy

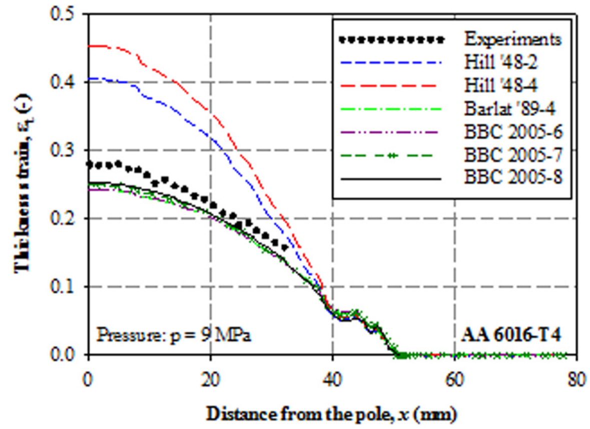


Fig. 2 Comparison between FE simulation and experiment for thickness-strain distribution

Figure 2 shows the distribution of the thickness strain versus the distance measured from the bulge axis (material coordinate before deformation). From this diagram, one may notice that the results provided by the BBC 2005-7 and BBC 2005-8 show the best agreement with the experimental data. The predictions of the yield criteria are very sensible to the number of input data. The results of the finite-element simulation are in the best agreement with the experimental data, when the whole set of eight input parameters is used [47].

In order to enhance the flexibility of the BBC2005 yield criterion, a new version (BBC2008) of this model has been developed [48]. The model is expressed as a finite series that can be expanded to retain more or fewer terms, depending on the amount of experimental data. Different identification strategies (using 8, 16, 24, etc., input values) could be used in order to determine the coefficients of the yield function.

The BBC2008 equivalent stress is defined as follows:

$$\frac{\bar{\sigma}^{2k}}{w-1} = \sum_{i=1}^{s}\left\{w^{i-1}\left\{\left[L^{(i)}+M^{(i)}\right]^{2k}+\left[L^{(i)}-M^{(i)}\right]^{2k}\right\}\right.$$
$$\left.+\,w^{s-i}\left\{\left[M^{(i)}+N^{(i)}\right]^{2k}+\left[M^{(i)}-N^{(i)}\right]^{2k}\right\}\right\}$$
$$k,s \in \mathbf{N}^{*}\ w=(3/2)^{1/s}>1$$
$$L^{(i)} = \ell_1^{(i)}\sigma_{11}+\ell_2^{(i)}\sigma_{22}$$
$$M^{(i)} = \sqrt{\left[m_1^{(i)}\sigma_{11}-m_2^{(i)}\sigma_{22}\right]^2+\left[m_3^{(i)}(\sigma_{12}+\sigma_{21})\right]^2}$$
$$N^{(i)} = \sqrt{\left[n_1^{(i)}\sigma_{11}-n_2^{(i)}\sigma_{22}\right]^2+\left[n_3^{(i)}(\sigma_{12}+\sigma_{21})\right]^2}$$
$$\ell_1^{(i)},\ell_2^{(i)},m_1^{(i)},m_2^{(i)},m_3^{(i)},n_1^{(i)},n_2^{(i)},n_3^{(i)} \in \mathbf{R} \quad (16)$$

The quantities denoted as $k, \ell_1^{(i)}, \ell_2^{(i)}, m_1^{(i)}, m_2^{(i)}, m_3^{(i)}, n_1^{(i)}, n_2^{(i)}, n_3^{(i)}$ $(i = 1,\ldots,s)$ are material parameters. One may prove that $k \in \mathbf{N}^*$ is a sufficient condition for the convexity of the yield surface defined by Eq. (14). The identification procedure to identify the coefficients is described in details in Refs. [6,48].

It is easily noticeable that Eq. (16) reduce to the isotropic formulation proposed by Barlat and Richmond [49], if

$$\ell_1^{(i)}=\ell_2^{(i)}=m_1^{(i)}=m_2^{(i)}=m_3^{(i)}=n_1^{(i)}=n_2^{(i)}$$
$$=n_3^{(i)}=1/2, \quad i=1,\ldots,s \quad (17)$$

Under these circumstances, the exponent k may be chosen as in Barlat and Richmond's model, i.e., according to the crystallographic structure of the sheet metal: $k = 3$ for BCC materials $(2k = 6)$ and $k = 4$ for FCC materials $(2k = 8)$.

Due to the expandable structure of the yield criterion, many identification strategies can be devised. In the papers [6] and [48], a procedure that uses only normalized yield stresses and r-coefficients obtained from uniaxial and biaxial tensile tests is presented. An identification procedure that strictly enforces a large number of experimental constraints on the yield criterion would be inefficient in practical applications. The failure probability of such a strategy increases when the external restrictions become stronger. Taking into account this aspect, the authors have developed an identification procedure based on the minimization of the error function [48].

Two versions of the BBC2008 yield criterion have been evaluated from the point of view of their performance [48]. They include 8 and 16 material coefficients, respectively, and

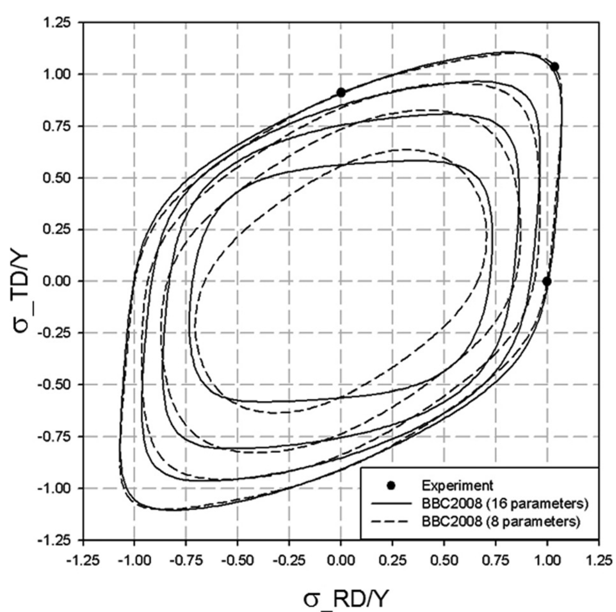


Fig. 3 Normalized yield surface predicted by BBC2008 model for AA2090-T3 aluminum alloy

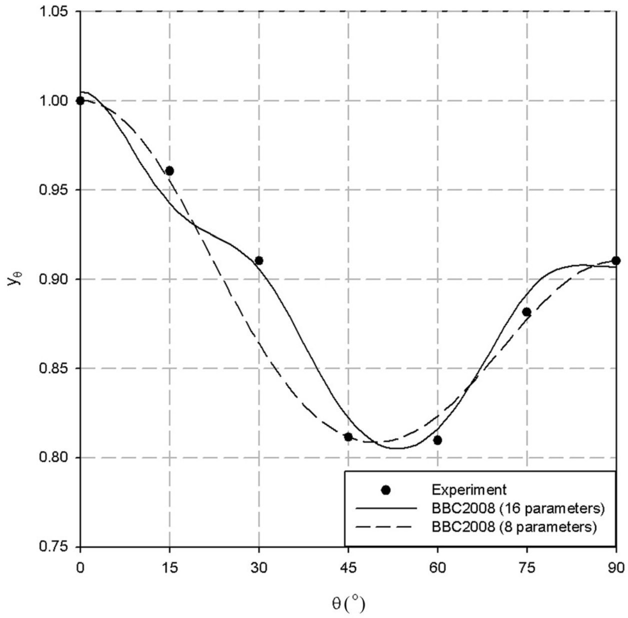


Fig. 4 Planar distribution of the uniaxial yield stress predicted by BBC2008 model for AA2090-T3 aluminum alloy

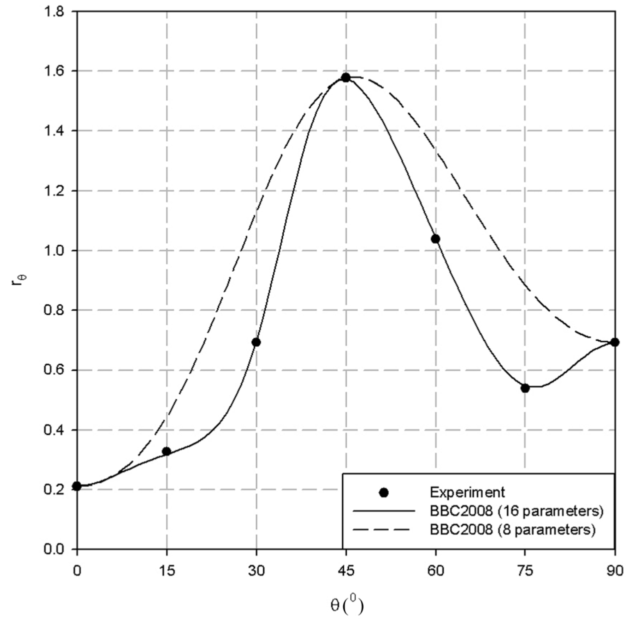
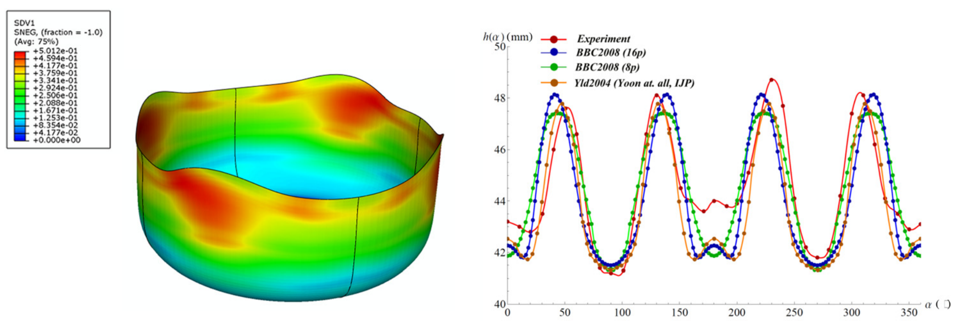


Fig. 5 Planar distribution of the *r*-coefficient predicted by BBC2008 model for AA2090-T3 aluminum alloy



(*a*) (*b*)

Fig. 6 Earing prediction for aluminum AA2090-T3: (*a*) simulation and (*b*) ears profile

correspond to the smallest values of the summation limit ($s = 1$ and $s = 2$). The identification of the BBC2008 (16 parameters) model has been performed using the following mechanical parameters: $y_{0\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{15\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{30\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{45\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{60\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{75\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{90\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{b}^{(\mathrm{exp})}$, $r_{0\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{15\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{30\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{45\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{60\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{75\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{90\,\mathrm{deg}}^{(\mathrm{exp})}$, and $r_{b}^{(\mathrm{exp})}$. In the case of BBC2008 (eight parameters), the input data have been restricted to the values $y_{0\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{45\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{90\,\mathrm{deg}}^{(\mathrm{exp})}$, $y_{b}^{(\mathrm{exp})}$, $r_{0\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{45\,\mathrm{deg}}^{(\mathrm{exp})}$, $r_{90\,\mathrm{deg}}^{(\mathrm{exp})}$, and $r_{b}^{(\mathrm{exp})}$.

The predictions of the BBC2008 model with 16 parameters are superior to those given by the eight-parameter version (see Figs. 3–5).

The improvement is noticeable especially in the case of the *r*-coefficients. This capability of the 16-parameter version is relevant for the accurate prediction of thickness when simulating sheet metal forming processes. For the materials exhibiting a distribution of the anisotropy characteristics that would lead to the occurrence of eight ears in a cylindrical deep-drawing process [50], the planar distribution of the *r*-coefficient predicted by the BBC2008 yield criterion with eight parameters is very inaccurate (see Ref. [48]). This model would not be able to predict the occurrence of more than four ears at the top edge of a cup deep-drawn from a circular blank. In contrast, the variation of the *r*-coefficient described by BBC2008 with 16 parameters closely follows the reference data. In conclusion, this model would predict the occurrence of six or eight ears as reported by Yoon et al. [32]. As compared with other formulations described in the literature, the new model does not use linear transformations of the stress tensor. Due to this fact, its computational efficiency should be superior in the simulation of sheet metal forming processes.

Figure 6(*a*) displays the simulated final geometry of deep drawn cup (for 16 parameters model) and the corresponding equivalent plastic strain distribution, whereas in Fig. 6(*b*), a comparison between predicted and experimental ears is given [50]. Also, for the sake of comparison, the ears' profile calculated with the Yld2004 model [32] is included.

Predictions of the BBC2008 model are in good agreement with the prediction of the Yld2004 model and also with the experiment [32]. As expected, the eight-parameter version was unable to predict six ears, which were experimentally observed. On the contrary, the 16-parameter version predicts six ears and their location, and at least qualitatively, the results are in good agreement with the experiment.

An extension of the BBC 2008 yield criterion has been proposed [51], which provides adaptive updates of the local anisotropy in the integration points of the macroscopic FE model. The BBC 2008 model is systematically recalibrated to the data provided by the crystal plasticity virtual experiment framework (VEF), using the advanced LAMEL model crystal plasticity

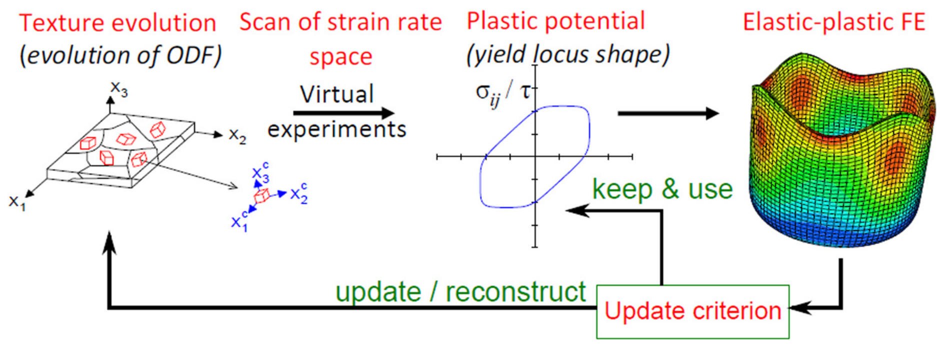


Fig. 7 HMS computational plasticity framework

model developed at the Catholic University Leuven [52]. An enhanced identification algorithm has been proposed. The new algorithm exploits comprehensive material characterization delivered by the VEF. A new hierarchical multiscale (HMS) framework that allows taking into account evolution of the plastic anisotropy during sheet forming processes has also been proposed (see Fig. 7). The earing number and height profile was measured experimentally for drawn cups for an AA6016 T4 aluminum alloy and compared to simulations with the continuously calibrated HMS-BBC2008 model (Fig. 8). The textures *A, B, C*, and *D* denote texture at 0%, 25%, 50% depths in thickness of sheet, and the average of the three, respectively. "Mechanical testing" denote the predicted cup profile using BBC2008 yield criterion identified using the results of the mechanical testing.

As it can be seen in Fig. 8, the HMS-BBC2008 simulations started from different textures tend toward decreasing ear height. With the only exception for the simulation initialized with the midthickness texture, the calculated cup profiles nearly coincide. Moreover, in terms of the ear height the predictions started from textures *A*, *B*, and *D* are accurate. This indicates that the selection of the initial texture is important but not necessarily predetermines the deformation process and can be spontaneously corrected by the local deformation-informed crystal plasticity code.

Theoretical and Experimental Determination of the Forming Limit Curves

Several theoretical models and a new experimental method to determine the limit strains have been developed in the CERTETA research center (presented below).

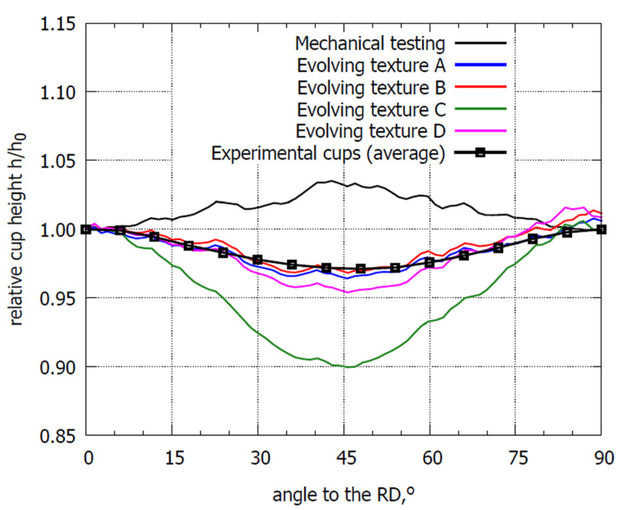


Fig. 8 Comparison of experimental and predicted cup profiles using BBC 2008 model identified by mechanical testing and using the evolving anisotropy HMS-BBC 2008 (Reprinted from Gawad et al., 2015 [51] with permission from Elsevier)

It is well known that the position and shape of the forming limit diagram (FLD) is influenced by the shape of the yield surface adopted in the computational model [53,54]. A sensitivity analysis regarding laws upon the limit strains is needed in the preprocessing stage. Such an analysis is also useful for the sheet metal producers, when trying to obtain materials having desired formability characteristics. Aiming to meet these requirements, a software package named FORM-CERT able to calculate FLDs [55] has been developed in the CERTETA center. The program is based on the Marciniak–Kuczynski (M–K) model of the necking process. A useful facility offered by the program is the possibility to perform the sensitivity analysis both for the yield surface and the forming limit curves. The numerical results can be compared with experimental data, using the import/export facilities included in the program. The program may be incorporated in finite-element codes.

Recently, the CERTETA team used the Gurson's model with some recent extensions to model the porous material, following both the evolution of a homogeneous sheet and the evolution of the distribution of voids [56]. At each moment, the material is tested for a potential change of plastic mechanism, by comparing the stresses in the uniform region to those in a virtual band with a larger porosity. The main difference with the coalescence of voids in a bulk solid is that the plastic mechanism for a sheet admits a supplementary degree-of-freedom, namely, the change in the thickness of the virtual band. For strain ratios close to the plane-strain case, the limit analysis (LA) model predicts almost instantaneous necking, but in the next step, the virtual band hardens enough to deactivate the localization condition. In this case, a supplementary condition for incipient necking has been applied, similar to the one used in Hill's model for the second quadrant. It has been showed that this condition is precisely the one for incipient bifurcation inside the virtual (and weaker) band. Figure 9 compares again the results of the new LA necking model and M–K

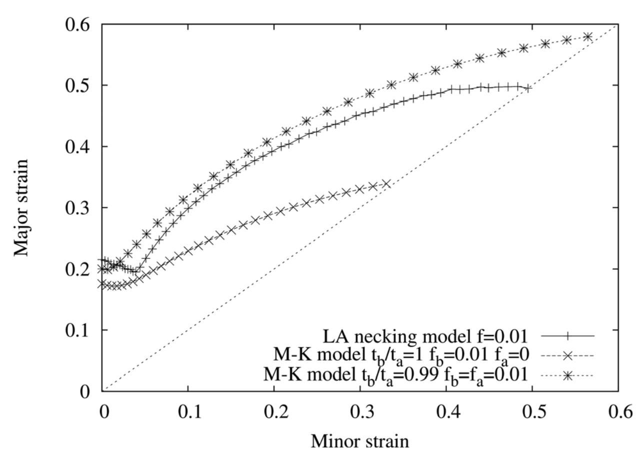


Fig. 9 Numerical FLD predictions for Gologanu model: LA necking model versus M–K model (Reprinted from Gologanu et al., 1913 [57] with permission from AIP)

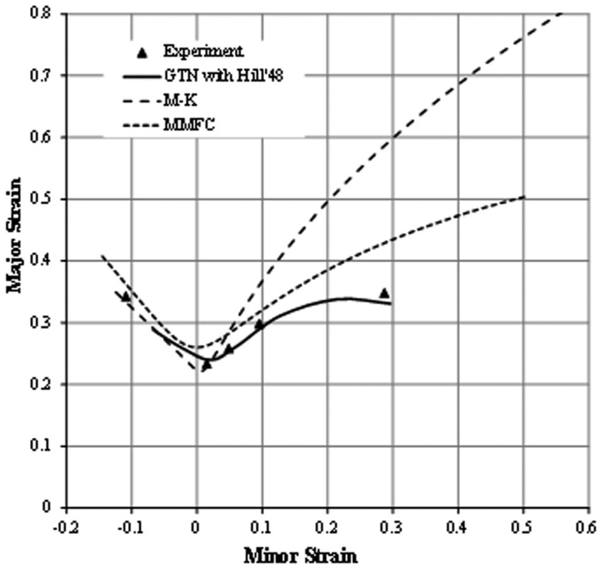


Fig. 10 Comparison between the FLC obtained by different methods (Reprinted from Kami et al., 2015 [57] with permission from Elsevier)

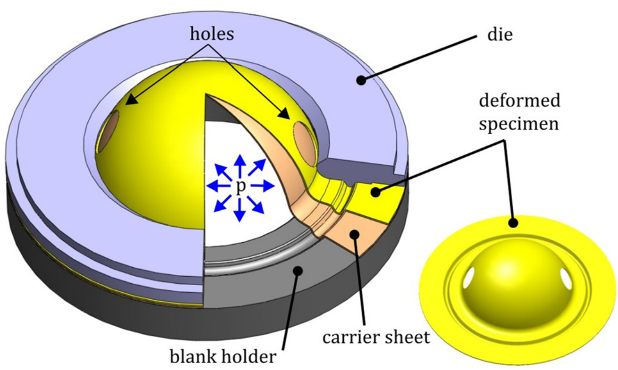


Fig. 11 Schematic view of the new formability test (Reprinted from Banabic et al., 2013 [59] with permission from Elsevier)

models. The following notations have been used: f-porosity in the LA model; f_a and f_b are the porosities in zones "a" and "b" in the MK model.

The Gurson–Tvergaard–Needleman (GTN) damage model has been used to determine the forming limit curve (FLC) of AA6016-T4 aluminum alloy [57] (see the mechanical parameters in page 4). Figure 10 indicates that the results obtained by numerical simulation using the GTN damage model are in good agreement with the experimental data. The comparison becomes even more favorable when confronted with the predictions of the Marciniak–Kuczynski (M–K) model and the modified maximum force criterion (MMFC) [58]—see Fig. 10. The Hill'48 yield criterion has been used in the FLC predictions. One may notice from the diagram that the quality of the GTN predictions is far better, especially along the right branch of the forming limit curve, where both M–K and MMFC models overestimate the formability of the metallic sheet. Fixing this deficiency can be achieved by implementing in the MK and MMFC plasticity models of nonquadratic criteria.

The CERTETA team developed a new procedure for the experimental determination of the FLCs [59]. The methodology is based on the hydraulic bulging of a double specimen (Fig. 11). The upper blank has a pair of holes pierced in symmetric positions with respect to the center, while the lower blank acts both as a carrier and a deformable punch. By modifying the dimensions and

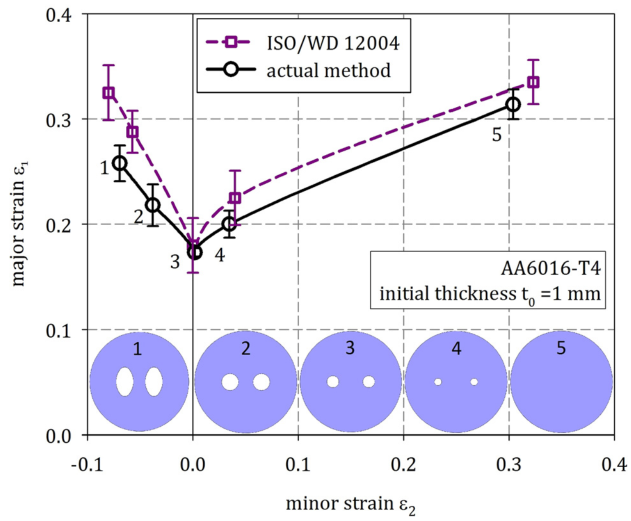


Fig. 12 Forming limit diagram of the AA6016-T4 alloy (Reprinted from Banabic et al., 2013 [59] with permission from Elsevier)

reciprocal position of the holes, it is possible to investigate the entire deformation range of the FLC. The most important advantages of the method proposed by the authors are the following: capability of investigating the whole strain range specific to the sheet metal forming processes; simplicity of the equipment; simplicity of the specimen configuration; reduction of the parasitic effects induced by the frictional interactions between the specimen and the other elements of the experimental device; and occurrence of the necking and fracture in the polar region of the specimen. The comparison between the FLCs determined using the new procedure and the Nakazima test shows minor differences. Figure 12 compares the FLCs obtained using the methodology proposed by the authors and the Nakazima test (according to the specifications of the international standard ISO 12004-2). In both cases, the limit strains have been measured using the ARAMIS (commercial system based on digital image correlation developed by GOM company from Germany) system.

Conclusion

The accuracy of the simulation results is given mainly by the accuracy of the material models. As it has been shown in the previous chapters, advanced yield criteria allow accurate prediction of the anisotropic behavior of materials. On the one hand, it is possible to simultaneously describe both the uniaxial yield stress variation and the anisotropic coefficient in the sheet. On the other hand, it is also possible to model both "first and second order anisotropic behavior anomalies." Furthermore, the yield criteria have also been extended to 3D.

The new yield criteria developed in the last years in the CERTETA research center show a very good prediction of the plastic anisotropy of sheet metals, especially for aluminum alloys. Comparison with data shows that the new criteria presented can successfully describe anisotropic behavior in both aluminum and steel sheets. In general, these models lead to yield surface shapes consistent with those predicted using polycrystal models. The biaxial yield stress and the biaxial anisotropy coefficient of the sheet metal are the parameters used in the identification procedure in the above-mentioned criteria. As shown by the results presented in this paper, the BBC2005 and BBC2008 yield criteria offer more accurate predictions than the classical yield criteria. The new models for FLC developed by the CERTETA team show a very good prediction of the experiments. The experimental procedure proposed to determine the limit strains demonstrates several advantages compared with the classical procedures.

In the future, the research in this field of study will be oriented toward developing new models that include special properties (superplastic materials, shape memory materials, etc.). By including the evolution of the coefficients in yield functions, it will be possible to predict the yield loci for nonlinear loading [60,61]. Stochastic modeling will be used for a more robust prediction of the yield loci and forming limit curves (taking into account the variability of the mechanical parameters). Coupling of the phenomenological models with the ones based on crystal plasticity will allow better simulation of the parameters' evolution in technological processes (temperature, strain rate, strain path, and structural evolution).

References

[1] Banabic, D., 2000, "Anisotropy of Sheet Metal," *Formability of Metallic Materials*, D. Banabic, ed., Springer Verlag, Heidelberg, Germany, pp. 119–172.

[2] Barlat, F., Banabic, D., and Cazacu, O., 2002, "Anisotropy in Sheet Metals," *Numerical Simulation of 3-D Sheet Metal Forming Processes*, D. Y. Yang, ed., Korea Advanced Inst. of Science and Technology (KAIST), Jeju Island, South Korea, pp. 515–524.

[3] Barlat, F., Cazacu, O., Zyczkowski, M., Banabic, D., and Yoon, J.-W., 2004, "Yield Surface Plasticity and Anisotropy," *Continuum Scale Simulation of Engineering Materials*, D. Raabe, L.-Q. Chen, F. Barlat, and F. Roters, eds., Wiley-VCH, Weinheim, Germany, pp. 145–185.

[4] Banabic, D., and Tekkaya, E. A., 2006, "Forming Simulation," *Virtual Fabrication of Aluminum Alloys*, J. Hirsch, ed., Wiley-VCH, Weinheim, Germany, pp. 275–302.

[5] Banabic, D., Barlat, F., Cazacu, O., and Kuwabara, T., 2007, "Anisotropy and Formability," *Advances in Material Forming-ESAFORM 10 Years On*, F. Chinesta and E. Cueto, eds., Springer, Heidelberg, Germany, pp. 143–173.

[6] Banabic, D., 2010, *Sheet Metal Forming Processes*, Springer, Heidelberg (in Chinese, Science Press, Beijing, 2015).

[7] Banabic, D., Barlat, F., Cazacu, O., and Kuwabara, T., 2010, "Advances in Anisotropy and Formability," Int. J. Mater. Form., **3**(3), pp. 165–189.

[8] Lee, M. G., and Barlat, F., 2014, "Modeling of Plastic Yielding, Anisotropic Flow and the Bauschinger Effect," *Comprehensive Materials Processing*, S. Hashmi, ed., Elsevier, Amsterdam, The Netherlands, pp. 235–260.

[9] Brosius, A., and Banabic, D., 2014, "Anisotropy," *Encyclopedia of Production Engineering*, L. Laperrière and G. Reinhart, eds., Springer, Heidelberg, Berlin, Germany, pp. 40–47.

[10] Banabic, D., 2014, "Fliessortkriteria," *Blechumformtechnik*, K. Siegert, ed., Springer, Heidelberg, Berlin, Germany.

[11] Bruschi, S., Altan, T., Banabic, D., Bariani, P. F., Brosius, A., Cao, J., Ghiotti, A., Khraisheh, M., Merklein, M., and Tekkaya, E., 2014, "Testing and Modeling of Material Behavior and Formability in Sheet Metal Forming Processes (Keynote Paper)," Ann. CIRP, **63**(2), pp. 727–749.

[12] Hill, R., 1948, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals," Proc. R. Soc. London, Ser. A, **193**(1033), pp. 281–297.

[13] Woodthrope, J., and Pearce, R., 1970, "Anomalous Behaviour of Aluminium Sheet Under Balanced Biaxial Tension," Int. J. Mech. Sci., **12**(4), pp. 341–347.

[14] Hill, R., 1979, "Theoretical Plasticity of Textured Aggregates," Math. Proc. Cambridge Philos. Soc., **85**(1), pp. 179–191.

[15] Hill, R., 1990, "Constitutive Modelling of Orthotropic Plasticity in Sheet Metals," J. Mech. Phys. Solids, **38**(3), pp. 405–417.

[16] Hill, R., 1993, "A User-Friendly Theory of Orthotropic Plasticity in Sheet Metals," Int. J. Mech. Sci., **35**(1), pp. 19–25.

[17] Hershey, A. V., 1954, "The Plasticity of an Isotropic Aggregate of Anisotropic Face Centred Cubic Crystals," ASME J. Appl. Mech., **21**, pp. 241–249.

[18] Hosford, W. F., 1979, "On Yield Loci of Anisotropic Cubic Metals," 7th North American Metalworking Conference, Dearborn, MI, pp. 191–197.

[19] Logan, R. W., and Hosford, W. F., 1980, "Upper-Bound Anisotropic Yield Locus Calculations Assuming Pencil Glide," Int. J. Mech. Sci., **22**(7), pp. 419–430.

[20] Barlat, F., and Lian, J., 1989, "Plastic Behavior and Stretchability of Sheet Metals. Part 1: A Yield Function for Orthotropic Sheets Under Plane Stress Conditions," Int. J. Plast., **5**(1), pp. 51–56.

[21] Gotoh, M., 1977, "Theory of Plastic Anisotropy Based on a Yield Function of Fourth Order (Plane Stress State)," Int. J. Mech. Sci., **19**(9), pp. 505–512.

[22] Budiansky, B., 1984, "Anisotropic Plasticity of Plane-Isotropic Sheets," *Mechanics of Material Behaviour*, G. J. Dvorak and R. T. Shield, eds., Elsevier, Amsterdam, The Netherlands, pp. 15–29.

[23] Tourki, Z., Makkouk, R., Zeghloul, A., and Ferron, G., 1994, "Orthotropic Plasticity in Sheet Metals," J. Mater. Processes Technol., **45**(1–4), pp. 453–458.

[24] Barlat, F., Lege, D. J., and Brenz, J. C., 1991, "A 6-Component Yield Function for Anisotropic Materials," Int. J. Plast., **7**(7), pp. 693–712.

[25] Barlat, F., Becker, R. C., Hayashida, Y., Maeda, Y., Yanagawa, M., Chung, K., Brem, J. C., Lege, D. J., Matsui, K., Murtha, S. J., and Hattori, S., 1997, "Yielding Description of Solution Strengthened Aluminum Alloys," Int. J. Plast., **13**(4), pp. 185–401.

[26] Barlat, F., Maeda, Y., Chung, K., Yanagawa, M., Brem, J. C., Hayashida, Y., Lege, D. J., Matsui, K., Murtha, S. J., Hattori, S., Becker, R. C., and Makosey, S., 1997, "Yield Function Development for Aluminum Alloy Sheets," J. Mech. Phys. Solids, **45**(11–12), pp. 1727–1763.

[27] Karafillis, A. P., and Boyce, M. C., 1993, "A General Anisotropic Yield Criterion Using Bounds and a Transformation Weighting Tensor," J. Mech. Phys. Solids, **41**(12), pp. 1859–1886.

[28] Bron, F., and Besson, J., 2004, "A Yield Function for Anisotropic Materials," Int. J. Plast., **20**(4–5), pp. 937–963.

[29] Barlat, F., Brem, J. C., Yoon, J. W., Chung, K., Dick, R. E., Lege, D. J., Pourboghrat, F., Choi, S.-H., and Chu, E., 2003, "Plane Stress Yield Function for Aluminium Alloy Sheets: Formulation," Int. J. Plast., **19**(9), pp. 1297–1319.

[30] Barlat, F., Aretz, H., Yoon, J. W., Karabin, M. E., Brem, J. C., and Dick, R. E., 2005, "Linear Transformation-Based Anisotropic Yield Functions," Int. J. Plast., **21**(5), pp. 1009–1039.

[31] Aretz, H., and Barlat, F., 2013, "New Convex Yield Functions for Orthotropic Metal Plasticity," Int. J. Non-Linear Mech., **51**, pp. 97–111.

[32] Yoon, J. W., Barlat, F., Dick, R. E., and Karabin, M. E., 2006, "Prediction of Six or Eight Ears in a Drawn Cup Based on a New Anisotropic Yield Function," Int. J. Plast., **22**(1), pp. 174–193.

[33] Cazacu, O., and Barlat, F., 2001, "Generalization of Drucker's Yield Criterion to Orthotropy," Math. Mech. Solids, **6**(6), pp. 613–630.

[34] Drucker, D. C., 1949, "Relation of Experiments to Mathematical Theories of Plasticity," ASME J. Appl. Mech., **16**, pp. 349–357.

[35] Cazacu, O., and Barlat, F., 2003, "Application of the Theory of Representation to Describe Yielding of Anisotropic Aluminum Alloys," Int. J. Eng. Sci., **41**(12), pp. 1367–1385.

[36] Liu, C., Huang, Y., and Stout, M. G., 1997, "On the Asymmetric Yield Surface of Plastically Orthotropic Materials: A Phenomenological Study," Acta Mater., **45**(6), pp. 2397–2406.

[37] Cazacu, O., Plunkett, B., and Barlat, F., 2006, "Orthotropic Yield Criterion for Hexagonal Close Packed Metals," Int. J. Plast., **22**(7), pp. 1171–1194.

[38] Vegter, H., Drent, P., and Huetink, J., 1988, "A Planar Isotropic Yield Criterion Based on Material Testing at Multiaxial Stress State," *Simulation of Materials Processing: Theory, Methods and Applications*, S. F. Shen and P. R. Dawson, eds., Balkema, Rotterdam, The Netherlands, pp. 345–350.

[39] Vegter, H., and van den Boogaard, A. H., 2006, "A Yield Function for Anisotropic Sheet Material by Interpolation of Biaxial Stress States," Int. J. Plast., **22**(3), pp. 557–580.

[40] Hill, R., 1950, *The Mathematical Theory of Plasticity*, Oxford University Press, Oxford, UK.

[41] Comsa, D. S., and Banabic, D., 2007, "Numerical Simulation of Sheet Metal Forming Processes Using a New Yield Criterion," Key Eng. Mater., **344**, pp. 833–840.

[42] Soare, S., Yoon, J. W., and Cazacu, O., 2008, "On the Use of Homogeneous Polynomials to Develop Anisotropic Yield Functions With Applications to Sheet Forming," Int. J. Plast., **24**(6), pp. 915–924.

[43] Banabic, D., Aretz, H., Comsa, D. S., and Paraianu, L., 2005, "An Improved Analytical Description of Orthotropy in Metallic Sheets," Int. J. Plast., **21**(3), pp. 493–512.

[44] Banabic, D., Comsa, D. S., and Balan, T., 2000, "A New Yield Criterion for Orthotropic Sheet Metals Under Plane Stress Conditions," 7th TPR 2000 Conference, May 11–12, D. Banabic, ed., Printek Publishing House, Cluj Napoca, Romania, pp. 217–224.

[45] Banabic, D., Kuwabara, T., Balan, T., Comsa, D. S., and Julean, D., 2003, "Non-Quadratic Yield Criterion for Orthotropic Sheet Metals Under Plane-Stress Conditions," Int. J. Mech. Sci., **45**(5), pp. 797–811.

[46] Banabic, D., and Sester, M., 2012, "Influence of Material Models on the Accuracy of the Sheet Forming Simulation," Mater. Manuf. Processes, **27**, pp. 304–308.

[47] Lăzărescu, L., Ciobanu, I., Nicodim, I., Comşa, D. S., and Banabic, D., 2013, "Effect of the Mechanical Parameters Used as Input Data in the Yield Criteria on the Accuracy of the FE Simulation of Sheet Metal Forming Processes," Key Eng. Mater., **554–557**, pp. 204–209.

[48] Comsa, D. S., and Banabic, D., 2008, "Plane-Stress Yield Criterion for Highly-Anisotropic Sheet Metals," Numisheet 2008 Conference, Interlaken, Switzerland, Sept. 1–5, pp. 43–48.

[49] Barlat, F., and Richmond, O., 1987, "Prediction of Tricomponent Plane Stress Yield Surfaces and Associated Flow and Failure Behaviour of Strongly Textured FCC Polycrystalline Sheets," Mater. Sci. Eng., **91**, pp. 15–29.

[50] Vrh, M., Halilovič, M., Starman, B., Štok, B., Comsa, D. S., and Banabic, D., 2011, "Earing Prediction in Cup Drawing Using the BBC2008 Yield Criterion," AIP Conf. Proc., **1383**, pp. 142–150.

[51] Gawad, J., Banabic, D., Van Bael, A., Comsa, D. S., Gologanu, M., Eyckens, P., Van Houtte, P., and Roose, D., 2015, "An Evolving Plane Stress Yield Criterion Based on Crystal Plasticity Virtual Experiments," Int. J. Plast., **75**, pp. 141–169.

[52] Van Houtte, P., Saiyi, L., Seefeld, M., and Delannay, L., 2005, "Deformation Texture Prediction: From the Taylor Model to the Advanced Lamel Model," Int. J. Plast., **21**(3), pp. 589–624.

[53] Lian, J., Barlat, F., and Baudelet, B., 1989, "Plastic Behaviour and Stretchability of Sheet Metals: Effect of Yield Surface Shape on Sheet Forming Limit," Int. J. Plast., **5**(2), pp. 131–147.

[54] Paraianu, L., Comsa, D. S., Ciobanu, I. I., and Banabic, D., 2012, "Effect of the Constitutive Law on the Accuracy of Prediction of the Forming Limit Curves," Key Eng. Mater., **504–506**, pp. 77–82.

[55] Jurco, P., and Banabic, D., 2005, "A User-Friendly Programme for Calculating Forming Limit Diagrams," 8th ESAFORM Conference, D. Banabic, ed., The Publishing House of the Romanian Academy, Bucharest, Romania, pp. 423–427.

[56] Gologanu, M., Comsa, D. S., and Banabic, D., 1913, "Theoretical Model for Forming Limit Diagram Predictions Without Initial Inhomogeneity," AIP Conf. Proc., **1532**, pp. 245–253.

[57] Kami, A., Dariani, B. M., Vanini, A. S., Comsa, D. S., and Banabic, D., 2015, "Numerical Determination of the Forming Limit Curves of Anisotropic Sheet Metals Using GTN Damage Model," J. Mater. Process. Technol., **216**, pp. 472–483.
[58] Hora, P., Tong, L., and Berisha, B., 2013, "Modified Maximum Force Criterion, a Model for the Theoretical Prediction of Forming Limit Curves," Int. J. Mater. Form., **6**(2), pp. 267–279.
[59] Banabic, D., Lazarescu, L., Paraianu, L., Ciobanu, I., Nicodim, I., and Comsa, D. S., 2013, "Development of a New Procedure for the Experimental Determination of the Forming Limit Curves," Ann. CIRP, **62**(1), pp. 255–258.
[60] Barlat, F., Gracio, J. J., Lee, M.-G., Rauch, E. F., and Vincze, G., 2014, "An Alternative to Kinematic Hardening in Classical Plasticity," Int. J. Plast., **27**(9), pp. 1309–1327.
[61] Barlat, F., Vincze, G., Grácio, J. J., Lee, M.-G., Rauch, E. F., and Tomé, C. N., 2014, "Enhancements of Homogenous Anisotropic Hardening Model and Application to Mild and Dual-Phase Steels," Int. J. Plast., **58**, pp. 201–218.